DOI: 10.6702/ijbi.201806_13(2).0003

Optimizing Expected Profit Under Flexible Allocation of Campaign Days and Budget for Management of Shopping Centers: A Machine Learning Approach

Nora Sharkasi

Tokyo International University, International University of Japan

Corresponding Author: nora.sharkasi@gmail.com

Jay Rajasekera

Institute of International Strategy, Tokyo International University

Ushio Sumita

Graduate School of International Management,
International University of Japan

ABSTRACT

In retail businesses, a sales campaign is typically organized over segments of consecutive days within a certain period so as to maximize total sales in that period. It is also common to design a sales campaign so that good-sales-days of the previous year would be designated as sales campaign days, with the expectation that the campaign effect could further enhance the potential of good-sales-days. Sharkasi, Sumita, and Yoshii (2015) showed that, contrary to these common practices, expected total sales can be increased by reallocating sales campaign days in a more flexible manner. Their paper, however, focused exclusively on expected total sales without regard to expected profit and did not consider the sales campaign budget. The purpose of the current paper is to fill this gap by incorporating the sales campaign budget per day, where the expected total sales would be increased as a concave function of the budget increment. Furthermore, expected profit rather than total expected sales is optimized. Based on a real dataset from a shopping center in Tokyo, this study shows that expected profit could be increased considerably with no additional cost using the optimization approach.

Keywords: Budget constraints, expected profit maximization, marketing flexibility, sales campaigns, shopping centers

1. INTRODUCTION

A shopping center (denoted by SC, hereafter) has many stores within its premises, providing a variety of services such as retail stores, restaurants, food services, beauty salons, and travel agencies. The origins of today's SCs can be traced to three SCs in the United States: Country Club Plaza, which opened in 1922 in Kansas City, Kansas; Town and Country Shopping Center, which opened in 1948 in Columbus, Ohio; and North Gate Shopping Center, which opened in 1948 in Seattle, Washington. Since then, SCs have expanded rapidly all over the world. According to *Guinness Book of World Records*, the largest SC in the world in terms of total area is the Dubai Mall, located within downtown Dubai, UAE. Construction took about four years. Now, the mall comprises 1,200 retail outlets and more than 160 food and beverage stores. Dubai Mall also includes an aquarium, an Olympic-size ice rink, and a 22-screen Cineplex. The store mix of many regional and national SCs are more or less similar (Burns & Warren, 1995; Wakefield & Baker, 1998).

The competitive advantage of SCs over individual, independent retail stores is that they offer a variety of stores and services in one place for the convenience of consumers. They can provide cost-performance efficiency for their business partners by allowing them to share parking lots, loading and unloading depots, and other related facilities. Today, however, SCs operate in an increasingly competitive environment characterized by overcapacity and a declining number of customers (Shim & Eastlick, 1998; LeHew & Fairhurst, 2000).

At the time of building a new SC, the decision about its location and size would be crucial to gain a competitive advantage over other existing SCs (Ahmadi-Javid, et al., 2018; Eckert, et al., 2015; Yu, et al., 2007; Christaller, 1966; Weber, 1969; Eppli & Benjamin, 1994). Other important decisions include selection of tenant stores and choice of so-called "anchor" stores. Once these decisions are made, it may be virtually impossible to change them in a short-term period. After this stage, it then becomes important to manage available resources wisely to enhance profitability. Knowing how to organize sales promotions efficiently would be the key to success. This paper addresses this challenging issue.

In the era of big data analysis, it is now possible to collect massive amounts of data from tenants of an SC via a point-of-sales (POS) system so as to assess the effects of promotional activities. An extensive literature exists for analyzing consumer purchasing behavior based on POS data, including studies by Eugene (1997), Yada et al. (2006), Taguchi (2010), and Ishigaki et al. (2011), to name only a few. Little research has been done, however, on how to use POS data to manage SC business. Kumar, Shah, and Venkatesan (2006) addressed issues on how to evaluate customer lifetime value at the individual customer level so as to maximize SC profitability. An interesting case study of an SC in Iran was examined by Balaghar, Majidazar, and Niromand (2012), who assessed the effectiveness of promotional tools such as advertisement, sales promotion, public relations, and direct selling.

The topic of profit optimization in retail business can be approached from various perspectives. Retailers aim to maximize profit by considering potential revenues, purchase costs, diminishing profits for products, and opportunity costs for unfulfilled demand. Optimal profit may also be approached through optimal traffic assignment, cost structure, capacity maximization, and property structure. In recent studies, models were jointly developed to optimize decisions for product prices, display orientations, and shelf-space locations in a product category to optimize profits (e.g., Sun, et al., 2018; Hubner, et al., 2016; Katsifou, et al., 2014; Murray, et al., 2010).

A mixed-integer programming and nonlinear optimization model incorporating both essential in-store costs and space and cross-elasticities for profit maximization was developed by Irion, et al. (2012). In a recent paper, Hubner and Schaal (2017) developed a decision model integrating assortment and shelf-space planning by considering stochastic and space-elastic demand, and out-of-assortment and out-of-stock substitution effects in order to produce near-optimal results even for large-scale problems. Models incorporating optimal product assortments, along with promotional efforts and inventory and pricing models, have been discussed by a wide array of researchers (Rambha, et al., 2018; Hubner & Schaal, 2017; Heuvel & Wagelmans, 2017; Hubner, et al., 2016; Chapados, et al., 2014; Katsifou, et al., 2014; Wang & Li, 2012; Bhattacharjeea & Rameshb, 2000). Another stream of literature focuses on optimal scheduling such as workforce shift scheduling and pricing in the retail sector for profit optimization (Yong & Li, 2017; Kabak, et al., 2008).

A typical management arrangement of an SC-wide promotion is that retail outlets provide the promotion for an event in conjunction with mall management. Based on the analysis of a survey and three months of actual data, Parsons (2003) found that SC-wide promotions are the most preferred. Furthermore, a combination of general entertainment and price-based promotions were found to be strong drivers for visits and spending by consumers, whereas community and educational promotions enhanced the traffic of non-customers. An SC-wide sales campaign is typically organized over segments of consecutive days. It is common to design a sales campaign in such a way that strong-sales-days of the previous year would be designated as sales campaign days for the current year. This is done because of the expectation that the campaign effect could further enhance the potential for strong-sales-days.

In the original paper by Sharkasi, Sumita, and Yoshii (2015), a mathematical model was developed for maximizing the expected total revenue in the SC by reconfiguring how to schedule sales campaign days over a future period. The paper showed that, contrary to the common practices described above, it could be more effective to schedule sales campaign days over weak-sales-days. This finding implies that the campaign effect to enhance expected total sales over weak-sales-days could be larger than that over strong-sales-days.

The purpose of the current paper is to expand on the original paper by Sharkasi, Sumita, and Yoshii (2015) by considering the problem of optimizing total expected profit rather than total expected sales, subject to a campaign budget.

This paper is structured in five sections. Section 2 focuses on data description and cleansing; Section 3 discusses problem formulation; Section 4 presents the numerical results of the study; and Section 5 provides study conclusions

2. DATA DESCRIPTION AND CLEANSING

In this study, we work on a set of real data obtained from an SC in Tokyo for Winter 2010 (December 2009 and January and February 2010) and for Winter 2011 (December 2010 and January and February 2011). The set of days in Winter 2010 is denoted by D_{LD} , where *LD* stands for *Learning Data*. Similarly, D_{TD} denotes the set of days in Winter 2011, to be used as *Testing Data*. Since the data structure of Winter 2011 is identical to that of Winter 2010, we describe the data structure only for day $i \in D_{LD}$. A record for day $i \in D_{LD}$ consists of the following elements.

$I_{CAMP}(i)$: the campaign flag indicating whether the i-*th* day was under a sales campaign

($I_{CAMP}(i)$=1) or not ($I_{CAMP}(i)$=0)

$s(i)$: the total sales of the i-*th* day in Japanese yen for the entire SC (2.1)

$t(i)$: the number of purchase transactions of the i-*th* day for the entire SC

It should be noted that, in the actual practice of the SC represented by the given dataset, two separate price-based promotional campaigns were organized in both Winter 2010 and Winter 2011. The indicator function $Win\_1(i)$ describes whether or not the i-*th* day was under the first campaign. The second indicator function, $Win_{2(i)}$, is defined similarly for the second campaign. It should be noted that $I_{CAMP}(i) = Win\_1(i) + Win\_2(i)$.

Table 1 shows the organization of the sales campaign days in Winter 2010 and Winter 2011. We note that December and January each have 31 days, whereas February has 28 days in both Winter 2010 and Winter 2011. The number of campaign days in each winter totals 36.

Table 1
Number of Sales Campaign Days During Winter 2010 and Winter 2011 (As Indicated by Shopping Center)

Winter 2010			
Start Date	**End Date**	**Campaign**	**# of Days**
December 1, 2009	December 28, 2009	*Win_1*	28
December 29, 2009	January 3, 2010	No campaign	5
January 4, 2010	January 11, 2010	*Win_2*	8
January 12, 2010	February 28, 2010	No campaign	47
Winter 2011			
Start Date	**End Date**	**Campaign**	**# of Days**
December 1, 2010	December 28, 2010	*Win_1*	28
December 29, 2010	January 3, 2011	No campaign	5
January 4, 2011	January 11, 2011	*Win_2*	8
January 12, 2011	February 28, 2011	No campaign	47

Throughout each year, the administration of the SC organizes some activities or special events to attract more visitors. Consequently, these activities may result in outliers in $s(i)$ and $t(i)$. More specifically, let μ_S and σ_S be the mean and the standard deviation of the total sales over the winter period under consideration, and define μ_T and σ_T similarly for the number of total purchase transactions. Outliers of $s(i)$ and $t(i)$ are then defined as follows.

$$s(i) \text{ is an outlier} \quad \Leftrightarrow \quad s(i) \geq \mu_S + 2\,\sigma_S$$

$$t(i) \text{ is an outlier} \quad \Leftrightarrow \quad t(i) \geq \mu_T + 2\,\sigma_T$$

Let $\mu_{S:\neg o}$ and $\mu_{S:o}$ be the average total sales of non-outlier days and that of outlier days, respectively. $\mu_{T:\neg o}$ and $\mu_{T:o}$ are defined similarly. If $s(i)$ and $t(i)$ are judged to be outliers, they are adjusted according to this formula:

$$s(i) \leftarrow s(i) \times \mu_{S:\neg o} / \mu_{S:o}\ ;\ t(i) \leftarrow t(i) \times \mu_{T:\neg o} / \mu_{T:o} \quad (2.2)$$

Outliers may also result for other reasons. For example, a store in the SC under study provides facilities for cultural classes; e.g., flower arrangement and piano lessons. Monthly fees for such classes are paid on a fixed date of the month, generating outliers in total sales. Outliers of this sort are adjusted by eliminating the corresponding total sales and purchase transactions rather than using formula (2.2). Figure 1 shows the effect of cleansing all outliers in $s(i)$ for Winter 2010 and Winter 2011. The counterparts for $t(i)$ are depicted in Figure 2.

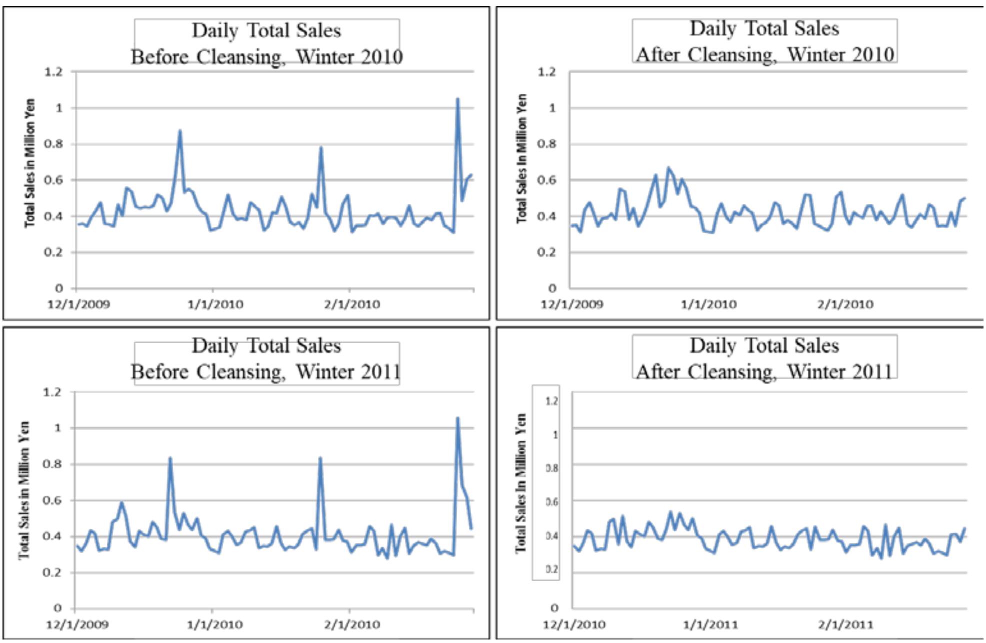

Figure 1. Total Sales Before and After Cleansing, Winter 2010 and Winter 2011

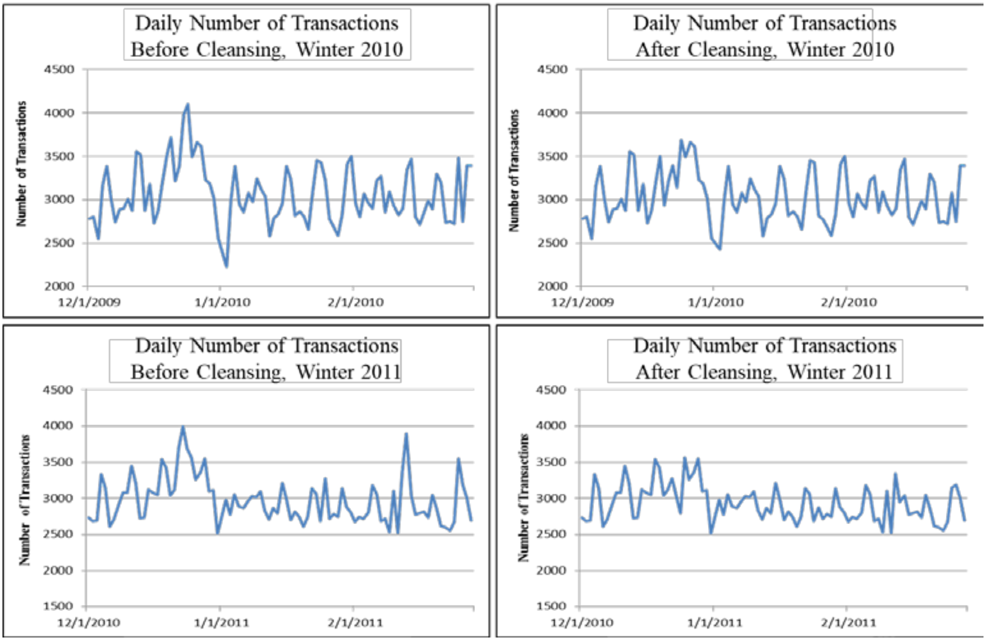

Figure 2. Purchase Transactions Before and After Cleansing, Winter 2010 and 2011

3. PROBLEM FORMULATION

In this section, we formulate the problem of determining the optimal allocation of campaign days and the campaign budget so as to maximize expected profit by expanding the mathematical model of Sharkasi, Sumita, and Yoshii (2015). Here, a promotional campaign is organized so as to maximize the expected profit over a given period of M days, subject to the number of campaign days being N and the campaign budget B_{Max}. A machine learning technique is used, with D_{LD} and D_{TD} as a set of *Learning Data* and a set of *Testing Data*, respectively. The total sales and the total number of purchase transactions for day $i \in D_{LD}$ are denoted by $s_{LD}(i)$ and $t_{LD}(i)$, respectively. For day $j \in D_{TD}$, $s_{TD}(j)$ and $t_{TD}(j)$ are defined similarly. The systematic approach used in this paper is structured in four steps, as discussed below.

3.1. Step I: Defining the Indicator Function of a Good-Sales-Day (GSD)

We begin our study by introducing the indicator function for a good-sales-day (GSD). This function is a composite of the key performance measures of promotional campaigns in shopping centers as guided by the literature; that is, the total sales $s_{LD}(i)$ and the total number of purchase transactions $t_{LD}(i)$ at the end of business day i. All days in D_{LD} are first placed in a descending order of $s_{LD}(i)$ and $t_{LD}(i)$, separately. The percentile points in $s_{LD}(i)$ and $t_{LD}(i)$ are then marked, which we denote as S_0 and T_0, respectively. Table 2 summarizes these values obtained from the real data for Winter 2010 (LD) at different percentile points.

Table 2

Values for Total Sales and Number of Purchase Transactions, Winter 2010 (LD)

Percentile	Total Sales in Billions (S_0)	Purchase Transactions (T_0)	Percentile	Total Sales in Billions (S_0)	Purchase Transactions (T_0)
10%	¥ 5.517	3,502	60%	¥ 4.021	2,946
20%	¥ 5.187	3,391	70%	¥ 3.876	2,870
30%	¥ 4.671	3,233	80%	¥ 3.591	2,803
40%	¥ 4.511	3,140	90%	¥ 3.459	2,733
50%	¥ 4.226	3,014	100%	¥ 3.093	2,227

Using the thresholds S_0 and T_0, the indicator function $I_{GOOD:S_0T_0}(i)$ for $i \in D_{LD}$ can be defined as

$$I_{GOOD:S_0T_0}(i) = \begin{cases} 1\ , & if\ s_{LD} \geq S_0\ and\ \ t_{LD}(i) \geq T_0 \quad , \\ 0\ , & else \end{cases} \tag{3.1}$$

where day i is considered to be a good-sales-day (GSD) if $I_{GOOD:S_0T_0}(i) = 1$.

The numerical thresholds S_0 and T_0 obtained from the learning data LD are used to identify GSDs in the testing data TD so that $I_{GOOD:S_0T_0}(j)$ can be defined similarly for $j \in D_{TD}$. As shown in Table 2, the values of S_0 and T_0 depend on which percentile is chosen. In the subsequent steps, a systematic approach is described to identify which threshold values of S_0 and T_0 best classify days as GSDs.

3.2. Step II: Using Logistic Regression and Confusion Matrix for Appropriate Segmentation of GSD

Given the indicator function for sales campaign days, $I_{CAMP}(i)$ for $i \in D_{LD}$ as defined in equation (2.1), a logistic regression model is developed to estimate the likelihood, $\rho_{GOOD}(j)$ of whether day $j \in D_{TD}$ in the future winter period is a GSD. For this purpose, we consider the set of explanatory variables given in Table 3.

Following the standard procedure for eliminating multi-collinearity, the correlation structure of these explanatory variables is given in Table 4. In this case, it happened that the correlation of every pair is less than 0.5 and no variables are eliminated because of multi-collinearity.

A logistic regression model is developed for estimating the likelihood, $\rho_{GOOD}(j)$, of whether or not day j in the future winter period is a GSD based on *LD*. Namely, from a set of the explanatory variables $x_k(i)$ for $i \in D_{LD}$ and $k = 1, \cdots, K$, let $\underline{x}(i) = [x_1(i), \cdots, x_k(i)]$, and $\underline{\beta} = [\beta_0, \beta_1, \cdots, \beta_K]$. We define $r\left(\underline{x}(i), \underline{\beta}\right)$ by

$$r\left(\underline{x}(i), \underline{\beta}\right) = \beta_0 + \sum_{k=1}^{K} \beta_k \cdot x_k(i) \quad . \tag{3.2}$$

The corresponding logistic regression model then yields the optimal coefficient vector $\underline{\beta}^*$, given by

$$\underline{\beta}^* = \arg\min_{\underline{\beta}} \sum_{i \in D_{LD}} \left\{ I_{GOOD:S_0T_0}(i) - \frac{e^{r(\underline{x}(i),\ \underline{\beta})}}{1 + e^{r(\underline{x}(i), \underline{\beta})}} \right\}^2 \quad . \tag{3.3}$$

If $\underline{x}(j)$ of day j in the future winter period is known, equation (3.3) enables one to assess the likelihood of day j being a GSD. This measure, denoted by $\rho_{GOOD}(j)$, can be computed as

$$\rho_{GOOD}(j) = \frac{e^{r(\underline{x}(j),\, \underline{\beta}^*)}}{1 + e^{r(\underline{x}(j),\, \underline{\beta}^*)}} \quad . \tag{3.4}$$

By specifying a threshold level ρ_{GOOD}, equation (3.4) then enables one to determine whether or not day j is judged to be a GSD. More specifically, we define

$$\hat{I}_{GOOD}(j) = \begin{cases} 1 \ , & if\ \rho_{GOOD}(j) \geq \rho_{GOOD} \\ 0 \ , & else \end{cases} \quad . \tag{3.5}$$

In order to determine the threshold level ρ_{GOOD}, we use the confusion matrix obtained, as shown in Table 5. This approach is widely used in the area of machine learning. Since $\underline{x}(j)$ is known for $j \in D_{TD}$, and $\rho_{GOOD}(j)$ can be computed from equation (3.4) above, it can be known that day j is a GSD when ($\hat{I}_{GOOD}(j) = 1$) or not ($\hat{I}_{GOOD}(j) = 0$). Consequently, we are in a position to see whether or not $\hat{I}_{GOOD}(j) = I_{GOOD:S_0T_0:TD}(j)$, yielding the confusion matrix as shown in Table 5.

Table 3

Definitions of Explanatory Variables Considered for Logistic Regression

Label	Description
Week $_{\_k}(i)$, *k=1, 2, 3, 4*	Each month has four weeks, labeled as: *Week_1*, *Week_2*, *Week_3*, and *Week_4*. Any week consists of seven days, where *Week_1* starts from the first day of the month. *Week_4* may include extra days until the end of the month. *Week_k*(i) =1 if day i belongs to Week_*k*; and 0, otherwise.
Weekday $_{\_k}(i)$, *k= 1 , ⋯ ,5*	Weekday_*k* (i) takes the value of 1 when day i in *Week_k* is a weekday; and 0 otherwise. Each week has five weekdays, Mon, Tue, Wed, Thu, and Fri, labeled as Weekday_1, Weekday_2, Weekday_3, Weekday_4, and Weekday_5, respectively.
Weekend $_{\_k}(i)$, *k= 1 , 2*	*Weekend* $_{\_k}(i)$ takes the value of 1 when day i in *Week_k* is Saturday or Sunday; and 0 otherwise.
National_Holiday (i)	This binary flag indicates that day i in *Week k* is an official national holiday in Japan.
Non_national_Holiday (i)	This binary flag indicates that day i is not an official national holiday but is likely to be very passive in business in Japan; e.g., December 28, 29, 30, and 31 when offices are typically closed.
Win_1 (i)	This binary variable takes the value of 1 if day i is under a sales campaign in December; and 0 otherwise.
Win_2 (i)	This binary variable takes the value of 1 if day i is under a sales campaign in January or February; and 0 otherwise.
LY_Transactions (i)	This binary variable takes the value of 1 if the number of transactions of day i of the last year was greater than or equal to T_0; and 0 otherwise. Here, the numbering of days of the last year is shifted forward so that day i of the last year has the same day of the week as day $i \in D_{LD}$. In our case, the number of transactions of the final day of the last year becomes unavailable because of this shifting. The missing value is obtained by taking the average of the values of the same day of the week within that month.

Table 4
Correlation Matrix of Variables Tested for Multi-Collinearity

	Week_1	*Week_2*	*Week_3*	*Week_4*	*Mon*	*Tue*	*Wed*	*Thu*	*Fri*	*Weekend*	*National Holiday*	*Non National Holiday*	*Win_1*	*Win_2*	*LY_Trans.*
Week_1	1														
Week_2	-0.304	1													
Week_3	-0.294	-0.304	1												
Week_4	-0.361	-0.372	-0.361	1											
Mon	0.022	0.011	0.022	-0.049	1										
Tue	0.003	-0.008	0.003	0.001	-0.165	1									
Wed	0.022	0.011	-0.057	0.023	-0.158	-0.165	1								
Thu	0.003	-0.008	0.003	0.001	-0.165	-0.173	-0.165	1							
Fri	-0.057	0.011	0.022	0.023	-0.158	-0.165	-0.158	-0.165	1						
Weekend	0.005	-0.012	0.005	0.001	-0.257	-0.270	-0.257	-0.270	-0.257	1					
National Holiday	-0.102	0.189	-0.102	0.011	0.108	-0.078	0.108	0.098	-0.075	-0.122	1				
Non-national Holiday	-0.118	-0.122	-0.118	0.328	0.072	0.063	0.072	0.063	-0.087	-0.141	-0.04	1			
Win_1	0.051	0.032	0.051	-0.122	-0.049	0.001	0.023	0.001	0.023	0.001	0.01	-0.145	1		
Win_2	0.206	0.194	-0.171	-0.210	0.105	-0.020	-0.010	-0.020	-0.010	-0.032	0.16	-0.069	-0.210	1	
LY_Trans.	-0.093	0.078	0.049	-0.032	-0.118	-0.078	-0.155	0.372	0.216	-0.186	0.02	-0.198	0.423	-0.001	1

Table 5
General Confusion Matrix

		Actual			
		Not GSD	**GSD**	**Total**	
Judgment	**Not GSD**	x_{00}	x_{01}	X_0	***Precision:***
	GSD	x_{10}	x_{11}	X_1	$\mathbf{x_{11}/X_1}$
	Total	Y_0	Y_1	X	
		Recall:	$\boldsymbol{x_{11}/Y_1}$	***Accuracy:***	$\boldsymbol{(x_{00}+x_{11})/X}$

The common measures for assessing the appropriateness of the selection of the best cut-off value ρ_{GOOD} are given by Recall = x_{11}/Y_1, Precision = x_{11}/X_1 and Accuracy = $(x_{00}+x_{11})/X$. Recall describes the portion of actual GSDs that were judged to be a GSD, whereas Precision is the portion of judged GSDs that were actually a GSD, and Accuracy represents the overall correctness of the judgment.

It is clear that Recall decreases while Precision increases as ρ_{GOOD} increases. In order to balance the two conflicting measures, we consider the optimization problem of maximizing Precision subject to Recall ≥ 0.75. This optimization problem is solved by varying ρ_{GOOD} with a stepwise of 0.01, yielding the best model with ${\rho^*}_{GOOD} = 0.64$, Precision* = 0.81, Recall* = 0.76, and Accuracy* = 0.82. This optimal threshold ${\rho^*}_{GOOD}$ corresponds to the percentile points $S_0^* = 3.591$ million and $T_0^* = 2{,}870$, representing the 80% and 70% levels of total sales and total number of purchase transactions in *LD*, respectively. The resulting confusion matrix of the best model is shown in Table 6.

The estimated regression coefficients and other statistical measures of the selected logistic regression model are summarized in Table 7.

Table 6

Confusion Matrix With $\rho^*{}_{GOOD} = 0.64$, $S_0^* = 3,886,266$ and $T_0^* = 2,949$

		Actual			
		$\neg\, GSD$	GSD	**Total**	
Judgment	$\neg\, GSD$	43	9	52	**Precision**
	GSD	7	29	36	**80.6%**
	Total	50	38	88	
		Recall	**76.3%**	**Accuracy**	**81.8%**

GSD: Good-Sales-Days, $\neg\, GSD$: Not Good-Sales-Days

Table 7

Estimated Coefficients of the Logistic Regression with $\rho^*{}_{GOOD} = 0.64$

$S_0^* = 3,886,266$ and $T_0^* = 2,949$

	Estimate	Std. Error	Sig
(Intercept)	-11.8941	5.1750	*
Weekend	3.588	1.1487	**
week1	-2.108	1.0065	*
week2	-2.059	0.9320	*
LY Transactions	3.582	1.1751	*
Non-national Holiday	3.326	1.5989	*
Win_1	1.879	0.8704	*
Win_2	3.133	1.2651	*

**: Confidence level of 5%; **: confidence level of 1%.*

Given the campaign day assignment, represented by $\underline{d} = [d(1), \cdots, d(j), \cdots, d(M)] \in \{0,1\}^M$, where

$$d(j) = \begin{cases} 1\ , & if\ day\ j\ is\ selected\ for\ sales\ campaign \\ 0\ , & else \end{cases}, \tag{3.6}$$

with $\sum_{j=1}^{M} d(j) \leq N$, and the explanatory variables for day j in Table 7, the logistic regression model can be used to define $\hat{I}_{GOOD}(j)$ as:

$$\hat{I}^{*}_{GOOD}(j) = \begin{cases} 1\ , & if\ \rho_{GOOD}(j) \geq \rho^{*}{}_{GOOD} \\ 0\ , & else \end{cases} \quad . \tag{3.7}$$

This in turn enables one to estimate the expected total sales per day in a future period, which is a vital step toward deciding the optimal campaign day assignment $\underline{d}^{*}$ and the optimal budget size B^{*} so as to maximize the expected total profit, as we discuss next.

3.3. Step III: Estimating Expected Total Sales per Day in D_{TD}

We now turn to the issue of how to estimate the expected total sales of day j, $j \in D_{TD}$ in the future winter period. For this purpose, we compute the average total sales, denoted by $\hat{s}_{(m,n)}$, over the learning period $i \in D_{LD}$ with $m = I_{CAMP}(i)$ and $n = I_{GOOD:S_0^* T_0^*}(i), m, n \in \{0,1\}$ as shown in Table 8, resulting in four values of the average total sales obtained from *LD;* that is, $\hat{s}_{(0,0)}$= ¥ 3.65 m, $\hat{s}_{(0,1)}$= ¥ 4.68 m, $\hat{s}_{(1,0)}$= ¥ 3.89 m, $\hat{s}_{(1,1)}$= ¥ 4.82 m (m: million).

Table 8

Average Total Sales Matrix $\hat{s}_{(m,n)}$ in ¥ Million Based on *LD*

		Good – Sales – Day	
		0	**1**
Campaign Day	**0**	$\hat{s}_{(0,0)}$	$\hat{s}_{(0,1)}$
	1	$\hat{s}_{(1,0)}$	$\hat{s}_{(1,1)}$

The expected total sales per day in D_{TD} should be estimated by taking into consideration the effect of the enhanced campaign budget. More specifically, let B_0 be the standard budget and define the enhanced campaign budget per day by $B = B_0 + \Delta_B$, where the budget increase Δ_B is examined with every increment of 10% of the standard budget B_0.

Two main settings are considered. First, if day $j \in D_{TD}$ in the future winter period under consideration is not chosen for the sales campaign (that is, if $d(j) = 0$), the campaign budget will not have an effect on the expected total sales of day j. Second, if day j is chosen for the sales campaign (that is, if $d(j) = 1$), it is natural to assume that the expected total sales would increase as Δ_B increases with the effect of diminishing return. This effect may depend on whether or not day j of D_{LD} was under the sales campaign. The effect of the former case is represented by $g_{CAMP \to CAMP}(\Delta_B)$ and that of the latter case by $g_{\neg CAMP \to CAMP}(\Delta_B)$, where both functions are strictly increasing and concave in Δ_B with $g_{CAMP \to CAMP}(0) = g_{\neg CAMP \to CAMP}(0) = 1$.

More specifically, let $\hat{r}_{(m,n) \to (m:_{\Delta_B}, n:_{\Delta_B})}(j)$ be the expected total sales of day j in D_{TD} estimated when day j in D_{LD} had $m = I_{CAMP}(j)$ and $n = I_{GOOD:\, S_0 T_0}(j)$, and day j in D_{TD} has $m:_{\Delta_B} = d(j)$, and $\hat{I}^*_{GOOD}\,(j)$. One then has:

$$\hat{r}_{(0,n) \to (0, n:_{\Delta_B})}(j) \quad = \hat{s}_{(0,n)}, \qquad n, n:_{\Delta_B} \in \{0,1\},$$

$$\hat{r}_{(0,n) \to (1, n:_{\Delta_B})}(j) \quad = \hat{s}_{(0,n)} \times g_{\neg CAMP \to CAMP}(\Delta_B)\,, \quad n,, n:_{\Delta_B} \in \{0,1\},\ \Delta_B > 0 \quad (3.8)$$

$$\hat{r}_{(1,n) \to (1, n:_{\Delta_B})}(j) \quad = \hat{s}_{(1,n)} \times g_{CAMP \to CAMP}(\Delta_B), \quad n,, n:_{\Delta_B} \in \{0,1\},\ \Delta_B > 0$$

Accordingly, the aggregated total expected sales, denoted by $\hat{R}(\underline{d}, \Delta_B)$, can be obtained as

$$\hat{R}(\underline{d}, \Delta_B) = \sum_{j=1}^{M} \sum_{m,n \in \{0,1\}} \hat{r}_{(m,n) \to (m:_{\Delta_B}, n:_{\Delta_B})}(j)\ \delta_{\{I_{CAMP}(i)=m\}} \delta_{\{I_{GOOD:\, S_0 T_0}(i)=n\}} \delta_{\{d(j)=m:_{\Delta_B}\}}\ \delta_{\{\hat{I}^*_{GOOD}\,(j)=n:_{\Delta_B}\}},$$

(3.9),

where $\delta_{\{STATEMENT\}} = 1$ if *STATEMENT* is true, and $\delta_{\{STATEMENT\}} = 0$ otherwise.

Before moving to the last step, we construct the two functions $g_{CAMP\rightarrow CAMP}(\Delta_B)$ and $g_{\neg CAMP\rightarrow CAMP}(\Delta_B)$ explicitly. For this purpose, we use a generic function $g(\Delta_B)$ of the form

$$g(\Delta_B) = 1 + \frac{a \cdot \Delta_B}{1 + b \cdot \Delta_B} \quad . \tag{3.10}$$

By differentiating $g(\Delta_B)$ twice with respect to Δ_B, one sees that

$$\frac{d}{dx}\, g(\Delta_B) = \frac{a(1 + b \cdot \Delta_B) - a\, b \cdot \Delta_B}{(1 + b \cdot \Delta_B)^2} = \frac{a}{(1 + b \cdot \Delta_B)^2} > 0 \tag{3.11}$$

and

$$\frac{d^2}{dx^2}\, g(\Delta_B) = -2b\, \frac{a}{(1 + b \cdot \Delta_B)^3} < 0 \quad , when \;\; \Delta_B > 0 \tag{3.12}$$

We note that $g(\Delta_B)$ is strictly increasing and concave in Δ_B. Furthermore, it can be seen that

$$\lim_{\Delta_B \to \infty} g(\Delta_B) = 1 + \frac{a}{b} \tag{3.13}$$

The functions $g_{CAMP\rightarrow CAMP}(\Delta_B)$ and $g_{\neg CAMP\rightarrow CAMP}(\Delta_B)$ can then be constructed by setting different values of a and b; that is,

$$g_{CAMP\rightarrow CAMP}(\Delta_B) = 1 + \frac{a_s \cdot \Delta_B}{1 + b_s \cdot \Delta_B} \;,$$
$$g_{\neg CAMP\rightarrow CAMP}(\Delta_B) = 1 + \frac{a_{\neg s} \cdot \Delta_B}{1 + b_{\neg s} \cdot \Delta_B} \,. \tag{3.14}$$

3.4. Step IV: Formulating the Optimization Problem for Maximizing Expected Profit

The optimization problem of expected profit, denoted by $\hat{P}(\underline{d}, \Delta_B)$, can be readily formulated as follows:

$$\hat{P}(\underline{d}^*, \Delta_B^*) = \max_{\underline{d}_{\Delta_B}, \Delta_B} \left[\hat{R}(\underline{d}, \Delta_B) - (B_0 + \Delta_B) \times \sum_{j=1}^{M} d(j) \right] \;, \tag{3.15}$$

subject to the campaign budget increase being $0 < B_0 + \Delta_B \leq B_{Max}$, and $\sum_{j=1}^{M} d_{\Delta_B}(j) \leq N$, where N is the actual number of sales campaign days organized over the learning data of Winter 2010.

4. NUMERICAL RESULTS FOR THE WINTER PERIOD

In order to define $g_{CAMP\to CAMP}(\Delta_B)$ and $g_{\neg CAMP\to CAMP}(\Delta_B)$ appropriately based on (3.14), we investigate the effect of varying two parameters a and b on the value of $g(\Delta_B)$ under two constraints, $a > b$ or $a < b$, with $0 < \Delta_B \leq 2\,B_0$. Figure 3(A) shows the case when a is kept constant at a=5 with varying parameter $b \in B = \{1, \cdots, 10\}$, and Figure 3(B) shows the case when b is kept constant at b=5 with varying parameter $a \in A = \{1, \cdots, 10\}$. Similar graphs are observed when a and b are fixed at different levels.

To determine (a_s, b_s) and $(a_{\neg s}, b_{\neg s})$, we first compute all possible combinations of $(a_s, b_s), (a_{\neg s}, b_{\neg s}) \in \{1, \cdots, 10\} \times \{1, \cdots, 10\}$ satisfying $a_s < b_s$ and $a_{\neg s} < b_{\neg s}$. Then, each of the resulting maximum total profits is compared against the actual total sales, minus the corresponding campaign cost. The appropriate choices of (a_s, b_s) and $(a_{\neg s}, b_{\neg s})$ are determined by selecting the ones that achieve the minimum absolute difference, resulting in $(a_s, b_s) = (9, 10)$ and $(a_{\neg s}, b_{\neg s}) = (6, 7)$. These parameter values are rather insensitive to the final maximum expected profit, as shown in equation (4.1). Furthermore, the sensitivity index *SI* is computed where $\hat{P}\left(\underline{d}^*, \Delta_B^*\right)_{MAX}$ is the maximum possible value of $\hat{P}\left(\underline{d}^*, \Delta_B^*\right)$ in (3.15) over all combinations of $(a_s, b_s), (a_{\neg s}, b_{\neg s}) \in \{1, \cdots, 10\} \times \{1, \cdots, 10\}$ satisfying $a_s < b_s$ and $a_{\neg s} < b_{\neg s}$ and $\hat{P}\left(\underline{d}^*, \Delta_B^*\right)_{MIN}$ is defined similarly for the minimum possible value, yielding:

$$SI = \frac{\hat{P}\left(\underline{d}^*, \Delta_B^*\right)_{MAX} - \hat{P}\left(\underline{d}^*, \Delta_B^*\right)_{MIN}}{\hat{P}\left(\underline{d}^*, \Delta_B^*\right)_{MAX}} = \frac{156.14 - 146.95}{156.14} = 0.059 \quad . \qquad (4.1)$$

The two functions $g_{CAMP\to CAMP}(\Delta_B)$ with $(a_s, b_s) = (9, 10)$ and $g_{\neg CAMP\to CAMP}(\Delta_B)$ with $(a_{\neg s}, b_{\neg s}) = (6, 7)$ are depicted in Figure 4(A) and 4(B).

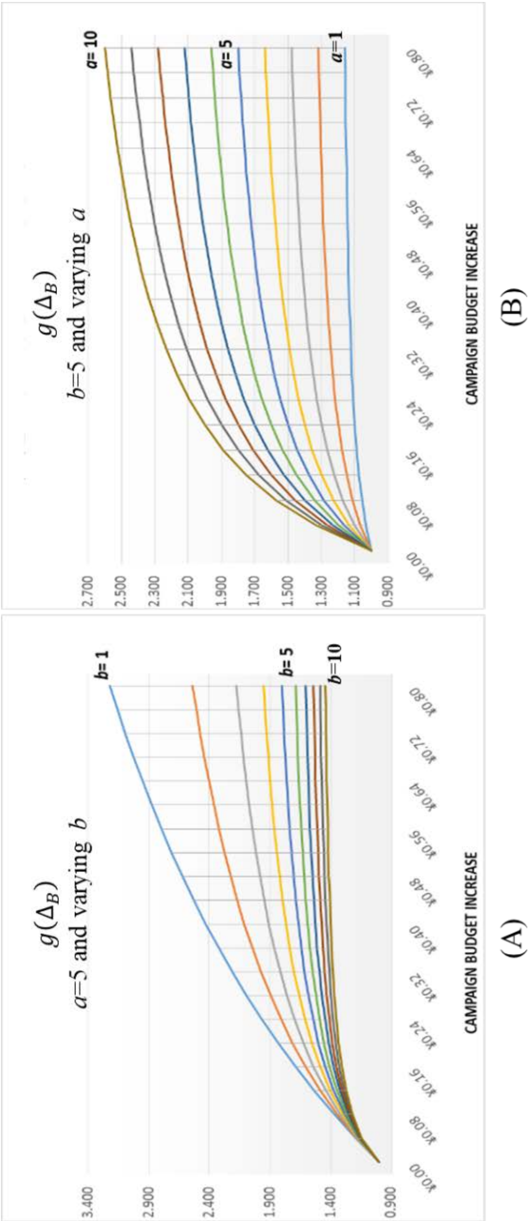

Figure 3(A) and 3(B). Behavior of $g(\Delta_B)$ for $0 < \Delta_B \leq 2B_0$

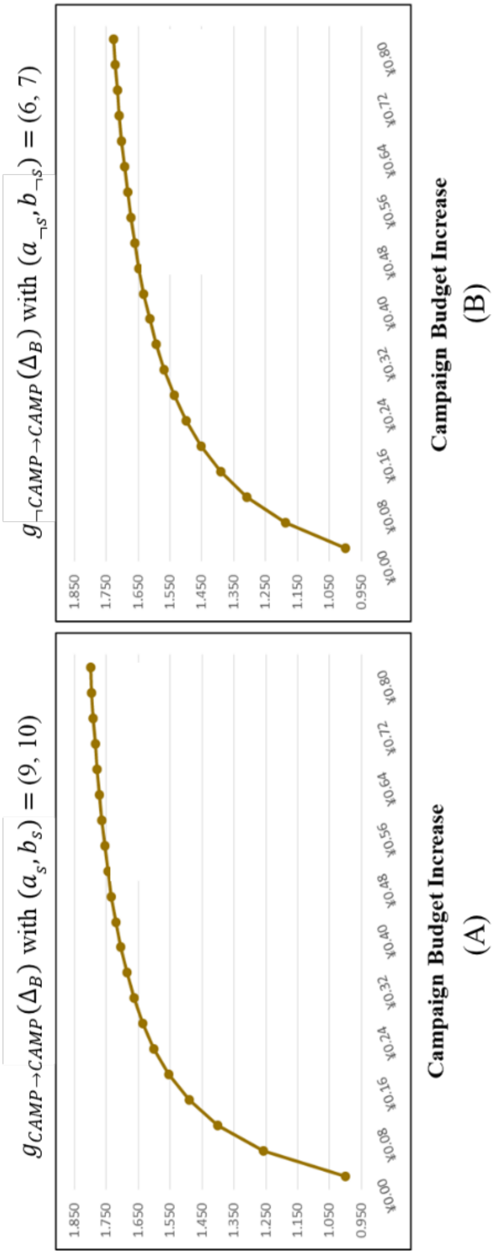

Figure 4(A) and 4(B). $g_{CAMP\to CAMP}(\Delta_B)$ **and** $g_{\neg CAMP\to CAMP}(\Delta_B)$ **with Appropriate** (a_s, b_s) **and** $(a_{\neg s}, b_{\neg s})$

With the standard campaign budget given by the shopping center to be $B_0 =$ ¥ 0.4 million per day, the expected total profit $\hat{P}(\underline{d}^*, \Delta_B)$ maximized over $\underline{d}$ given Δ_B per day is exhibited as a function of Δ_B, yielding the final optimal expected profit of $\hat{P}(\underline{d}^*, \Delta_B^*) =$ ¥ 151.49 million, with the optimal budget increase of $\Delta_B{}^* =$ ¥ 0.32 million (Figure 5).

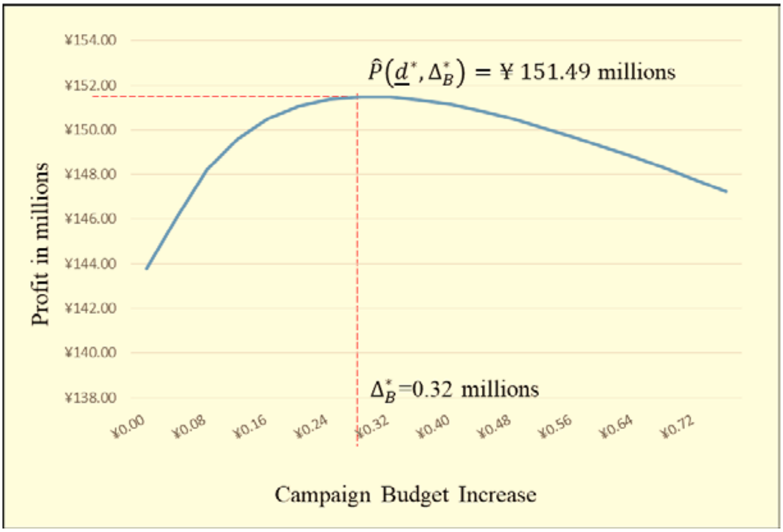

Figure 5. Expected Total Profit $\hat{P}(\underline{d}^*(\Delta_B), \Delta_B)$ Maximized over $\underline{d}$ Given Δ_B

The difference between the actual total sales minus the corresponding sales campaign cost obtained from the real data and $\hat{P}(\underline{d}^*, \Delta_B^*) =$ ¥ 151.49 million is compared with the actual profit $P(\underline{I}_{CAMP}) =$ ¥ 129.7 million, yielding a 16.8% increase in expected profit by reallocating 23 sales campaign days over the winter period with an optimal sales campaign budget per day of $B^* =$ ¥ 0.72 million (Table 9).

Table 9

Actual and Optimal Profits under Increased Budget for Winter 2011 (*TD*)

	$\boldsymbol{B_0}$	$\sum_{j=1}^{M} I_{CAMP}(j)$	$\boldsymbol{B_0} \times \sum_{j=1}^{M} I_{CAMP}(j)$	$\boldsymbol{R}(\underline{\boldsymbol{I}}_{\boldsymbol{CAMP}})$	$\boldsymbol{P}(\underline{\boldsymbol{I}}_{\boldsymbol{CAMP}})$	**Increase from Actual Profit** $\boldsymbol{P}(\underline{\boldsymbol{I}}_{\boldsymbol{CAMP}})$
Actual Profit $\boldsymbol{P}(\underline{\boldsymbol{I}}_{\boldsymbol{CAMP}})$	¥ 0.40	36	¥ 14.4	¥ 144.10	¥ 129.7	
Under Increased Budget $\widehat{\boldsymbol{P}}(\underline{d}^*_{\Delta_B}, \Delta_B{}^*)$	$(\boldsymbol{B_0} + \Delta_B^*)$	$\sum_{j=1}^{M} d^*(j)$	$(\boldsymbol{B_0} + \Delta_B^*) \times \sum_{j=1}^{M} \underline{d}^*$	$\widehat{\boldsymbol{R}}^*(\underline{\boldsymbol{d}}, \Delta_{\boldsymbol{B}})$	$\widehat{\boldsymbol{P}}(\underline{d}^*, \Delta_B{}^*)$	**16.8%**
	¥ 0.72	23	¥ 16. 56	¥ 168.05	¥ 151.49	

Table 10 presents the number of sales campaign days (*SCDs*) in actual practice and the optimal solution. The optimal solution $\hat{P}\left(\underline{d}^*, \Delta_B^*\right)$ is achieved by allocating 7 and 2 days in December and January, respectively, and 14 days in February, compared with 0 in actual practice.

Table 11 indicates how GSDs were optimally allocated compared with actual practice. One observes that 42 days were judged as GSDs under the optimal decision, compared with 34 GSDs in actual practice, which amounts to a 23.5% (8/34) increase in GSDs in the optimal decision.

Table 10

Number of *SCDs* Allocated in Actual Practice and Under Optimal Solution, Winter 2011

Month	Actual Number of Campaign Days	Optimal Number of Campaign Days
December	28	7
January	8	2
February	0	14
Total	36	23

Table 11
Effect of Optimal Decision Approach on GSDs

# of GSDs		GSD in Optimal Solution		Total
		1	**0**	
GSD in Actual Practice	**1**	31	3	34
	0	14	40	54
		45	43	88

Table 12 presents further details on how the above improvement by the optimal decision approach was achieved, where GSD versus ¬GSD transitions are classified according to sales campaign days only in actual practice, those in common, and those only by the optimal decision approach.

In actual practice, 15+1=16 were assigned as GSDs [that is, 44% (16/36)] and 55.5% (20/36) were assigned as ¬GSD. In contrast, the optimal decision approach allocated only 8+1=9 days (or 39%) to GSDs in actual practice and 11+3 = 14 days (or 60%) to¬GSD. This result supports the original observation that the effect of a sales campaign on enhancing the total sales of ¬GSD may exceed that of strengthening the total sales of GSD further. This result is also consistent with that reported in Sharkasi, Yoshi, and Sumita (2015).

Table 12

GSD vs. ¬GSD Transitions by Optimal Decision

Actual Only

SCD		Optimal GSD	Optimal ¬GSD	Total
Actual	GSD	12	3	15
Actual	¬GSD	2	15	17
Total		14	18	32

In Common

SCD (Common)		Optimal GSD	Optimal ¬GSD	Total
Actual	GSD	1	0	1
Actual	¬GSD	2	1	3
Total		3	1	4

Optimal Only

SCD		Optimal GSD	Optimal ¬GSD	Total
Actual	GSD	8	0	8
Actual	¬GSD	10	1	11
Total		18	1	19

5. CONCLUSION AND DISCUSSION

An extensive literature exists concerning shopping centers and related sales optimization, where different approaches are taken; e.g., how to find the optimal allocation of SCs among available alternatives and how to determine the configuration of space and design so as to achieve either cost-performance efficiency or profit generation. To the best of our knowledge, however, the problem of optimally allocating campaign days over a certain period (e.g., the winter season) has not been addressed in the literature. This paper is an extension of Sharkasi et al. (2015). It aimed to investigate the effect of flexible allocation of sales campaign days on expected total profit by incorporating the campaign budget per day as part of the optimization problem. For this purpose, the impact of budget increments on revenue was incorporated by defining a concave function to exhibit the effect of diminishing returns.

Through numerical examples involving actual data from a shopping center in Tokyo, the proposed model showed that expected profit can be maximized by optimal allocation of sales campaign days and campaign budget, achieving a 16.8% increase in expected profit with fewer sales campaign days by improving the sales campaign budget per day by 80%. The management of a shopping center can better serve its tenants by using this systematic approach to optimize the assignment and budgeting of sales campaign days.

With respect to limitations of the current study, the optimized profit in this study was gross profit and not net profit or net earnings because of limited data access from the shopping center.

It should be noted that this optimization problem took into consideration a constraint of a minimum number of days organized during the Christmas season in order to meet the realistic goals of an SC. It should also be noted that numerical results obtained from applying this systematic approach to the fall period yielded similar observations. Last, we note that, even though the dataset is not very recent, it could still be considered to solve the problem proposed in this paper because the standing managerial practice of SCs to schedule sales campaign days in the current year with reference to the previous year's schedule remains unchanged.

A possibility for future research could be optimizing net profit by further taking into consideration the possible impact of the proposed flexible assignment of sales campaign days on the marketing cost structure. Two assumptions could be laid forward. First, the flexible allocation of sales campaign days could exhaust the cost structure because of increased spending on advertising to reach out to prospects to inform them of possible fragmented assignment of sales campaign days. The second assumption could be quite contrasting. With the current pressure on brick-and-mortar SCs to reduce costs under intensifying competition from contemporary online and mobile shopping platforms, SCs may be able to leverage the power of smartphones to connect with prospects intelligently through GPS capability. This way, the SC could generate massive customized alerts with meager additional costs in the promise of maximizing revenue through optimal allocation of sales campaign days. Based on the managerial style of implementing the proposed approach of flexible allocation of sales campaign days, the optimization problem of net profit could be structured and solved accordingly.

ACKNOWLEDGEMENT

The authors would like to thank the anonymous reviewers for their kind suggestions that lead to further improve the paper.

REFERENCES

Ahmadi-Javid, A.; Amiri, E.; and Meskar, M. (2018). A profit-maximization location-routing-pricing problem: A branch-and-price algorithm, *European Journal of Operational Research* (in press; available online 25 February 2018). DOI: 10.1016/j.ejor.2018.02.020

Balaghar, A.; Majidazar M.; and Niromand M. (2012). Evaluation of effectiveness of sales promotional tools on sales volume – Case study: Iran tractor manufacturing complex (IT MC), *Middle-East Journal of Scientific Research 11* (4), 470- 480.

Bhattacharjee, S., and Ramesh, R. (2000). A multi-period profit maximizing model for retail supply chain management: An integration of demand and supply-side mechanisms, *European Journal of Operational Research 122*(3), 584-601. DOI: 10.1016/s0377-2217(99)00097-1

Burns, D.J., and Warren, H.B. (1995, 12). Need for uniqueness: Shopping mall preference and choice activity, *International Journal of Retail & Distribution Management 23*(12), 4-12. DOI: 10.1108/09590559510103954

Chapados, N.; Joliveau, M.; L'Ecuyer, P.; and Rousseau, L. (2014). Retail store scheduling for profit, *European Journal of Operational Research 239*(3), 609-624. DOI: 10.1016/j.ejor.2014.05.033

Christaller, W., and Baskin, C.W. (1966). *Central Places in Southern Germany*. Englewood Cliffs, NJ: Prentice-Hall.

Eckert, A.; He, Z.; and West, D.S. (2015). An empirical analysis of tenant location patterns near department stores in planned regional shopping centers, *Journal of Retailing and Consumer Services 22*, 61-70.
DOI: 10.1016/j.jretconser.2014.09.007

Eppli, M.J., and Benjamin, J.D. (1994). The evolution of shopping center research: A review and analysis, *Journal of Real Estate Research 9*(1), 5-32.

Flamand, T.; Ghoniem, A.; Haouari, M.; and Maddah, B. (2017). Integrated assortment planning and store-wide shelf space allocation: An optimization-based approach, *Omega* (in press; available online 21 October 2017). DOI: 10.1016/j.omega.2017.10.006

Groover, M.P. (1987). *Automation, Production Systems and Computer-Integrated Manufacturing*, Upper Saddle River, New Jersey: Prentice-Hall Publishers. DOI: https://doi.org/10.1108/aa.2002.22.3.298.2

Heuvel, W.V., and Wagelmans, A P. (2017). A note on 'A multi-period profit maximizing model for retail supply chain management,' *European Journal of Operational Research 260*(2), 625-630. DOI: 10.1016/j.ejor.2017.01.014

Hübner, A.; Kuhn, H.; and Kühn, S. (2016). An efficient algorithm for capacitated assortment planning with stochastic demand and substitution, *European Journal of Operational Research 250*(2), 505-520. DOI: 10.1016/j.ejor.2015.11.007

Irion, J.; Lu, J.; Al-Khayyal, F.; and Tsao, Y. (2012). A piecewise linearization framework for retail shelf space management models, *European Journal of Operational Research 222*(1), 122-136. DOI: 10.1016/j.ejor.2012.04.021

Jeffery, M. (2010). *Data-Driven Marketing: The 15 Metrics Everyone in Marketing Should Know*, New York: John Wiley Publishers. DOI: 10.1002/9781119198666

Jones, E. (1997). An analysis of consumer food shopping behavior using supermarket scanner data: Differences by income and location, *American Journal of Agricultural Economics 79*(5), 1437. DOI: 10.2307/1244358

Kabak, Ö.; Ülengin, F.; Aktaş, E.; Önsel, Ş.; and Topcu, Y.I. (2008). Efficient shift scheduling in the retail sector through two-stage optimization, *European Journal of Operational Research 184*(1), 76-90. DOI: 10.1016/j.ejor.2006.10.039

Katsifou, A.; Seifert, R.W.; and Tancrez, J. (2014). Joint product assortment, inventory and price optimization to attract loyal and non-loyal customers, *Omega 46*, 36-50. DOI: 10.1016/j.omega.2014.02.002

Kumar, V.; Shah, D.; and Venkatesan, R. (2006). Managing retailer profitability—one customer at a time! *Journal of Retailing 82*(4), 277-294.
DOI: 10.1016/j.jretai.2006.08.002

Lehew, M.L., and Fairhurst, A.E. (2000). U.S. shopping mall attributes: An exploratory investigation of their relationship to retail productivity, *International Journal of Retail & Distribution Management 28*(6), 261-279. DOI: 10.1108/09590550010328535

Muhlebach, R.F., and Alexander, A.A. (2019). *Shopping Center Management and Leasing*, Institute of Real Estate Management.

Murray, C.C.; Talukdar, D.; and Gosavi, A. (2010). Joint optimization of product price, display orientation and shelf-space allocation in retail category management, *Journal of Retailing 86*(2), 125-136. DOI: 10.1016/j.jretai.2010.02.008

Sharkasi,N.; Sumita, U.; and Yoshii, J. (2014). Development of enhanced marketing flexibility by optimally allocating sales campaign days for maximizing total expected sales, *Global Journal of Flexible Systems Management 16*(1), 87-95. DOI: 10.1007/s40171-014-0082-9

Parsons, A.G. (2003). Assessing the effectiveness of shopping mall promotions: Customer analysis, *International Journal of Retail and Distribution Management 31*(2), 74-79. DOI: 10.1108/09590550310461976

Rambha, T.; Boyles, S.D.; Unnikrishnan, A.; and Stone, P. (2018). Marginal cost pricing for system optimal traffic assignment with recourse under supply-side uncertainty, *Transportation Research Part B: Methodological 110*, 104-121. DOI: 10.1016/j.trb.2018.02.008

Shim, S., and Eastlick, M.A. (1998). The hierarchical influence of personal values on mall shopping attitude and behavior, *Journal of Retailing 74*(1), 139-160. DOI: 10.1016/s0022-4359(99)80091-8

Sun, W.; Zheng, J.; and Liu, H.X. (2018). A capacity maximization scheme for intersection management with automated vehicles, *Transportation Research Part C: Emerging Technologies.* DOI: 10.1016/j.trc.2017.12.006

Wakefield, K.L., and Baker, J. (1998). Excitement at the mall: Determinants and effects on shopping response, *Journal of Retailing 74*(4), 515-539. DOI: 10.1016/s0022-4359(99)80106-7

Wang, X., and Li, D. (2012). A dynamic product quality evaluation-based pricing model for perishable food supply chains, *Omega 40*(6), 906-917. DOI: 10.1016/j.omega.2012.02.001

Wang, Y., and Li, L. (2017). Manufacturing profit maximization under time-varying electricity and labor pricing, *Computers and Industrial Engineering 104*, 23-34. DOI: 10.1016/j.cie.2016.12.011

Weber, J. (1969). Evidence for discovery of gravitational radiation, *Physical Review Letters 22*(24), 1320-1324. DOI: 10.1103/physrevlett.22.1320

Yada, K.; Motoda, H.; Washio, T.; & Miyawaki, A. (2004). Consumer behavior analysis by graph mining technique, *Lecture Notes in Computer Science Knowledge-Based Intelligent Information and Engineering Systems*, 800-806. DOI:10.1007/978-3-540-30133-2_105

Yu, B., Yang, Z., & Cheng, C. (2007). Optimizing the distribution of shopping centers with parallel genetic algorithm. Engineering Applications of Artificial Intelligence, 20(2), 215-223. DOI: 10.1016/j.engappai.2006.06.015

ABOUT THE AUTHORS

Jay Rajasekera is Professor of Digital Business and Strategy at the Institute of International Strategy (IIS) at Tokyo International University and also serves as a vice president. He has co-authored four books, two of which are stand-alone entries. His professional career began in 1984 at Bell Laboratories in New Jersey, U.S.A, after earning his Ph.D. from North Carolina State University, U.S.A. He earned the Special Contribution Award twice from Bell Laboratories. One of his online scheduling systems is used for accepting multi-billion dollars of orders per year at Lucent Technologies. He has broad consulting experience with top information technology and telecommunications companies both in Japan and the U.S., as well as government organizations in Japan and other countries.

Ushio Sumita is Professor Emeritus at the University of Tsukuba, Japan. He currently teaches in the Graduate School of Business Administration, Keio University, Japan, and at the Graduate School of International Management at the International University of Japan. He also serves as a technical advisor for the Information Science Department, Information Systems Division, LIXIL Corporation, and for Matsuda Electronic Engineering Corporation. He received his first Ph.D. in 1981 from the University of Rochester, Rochester, New York, U.S.A., and his second Ph.D. in 1987 from Tokyo Institute of Technology, Japan. He has a broad range of research interests and has published more than 170 papers in leading archive journals in such areas as information technology, big data analytics, artificial intelligence, financial engineering, marketing, operations management, operations research, and organization theory. He has also conducted rigorous mathematical analyses in the areas of applied probability, stochastic processes, and queuing theory. Ushio has been involved in various consulting activities in numerous areas including IoT, big data analytics, new product development, logistics, value chain management, information technology, e-business, digital marketing, financial engineering, human resources management, and business process management.

Nora Sharkasi is currently an associate professor in digital marketing and entrepreneurship at Tokyo International University, International University of Japan. She obtained her bachelor’s degree from LaRoche College, U.S.A.; her master’s degree in analysis, design, and management of information systems from London School of Economics and Political Science LSE, United Kingdom; and her Ph.D. from the University of Tsukuba, Japan.